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混合式殘差置信傳播解碼器與基於檢查節點度數序列之極化碼稀疏圖列表解碼器

Hybrid RBP and Check-Node Degree Sequence Based Sparse Graph List Decoding of Polar Codes

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Introduction

Polar Codes are adopted as the core channel coding scheme in fifth-generation (5G) mobile communication technology, offering theoretical performance close to the Shannon limit. Converting the Polar Code structure into an LDPC-like code and utilizing the highly parallel Belief Propagation (BP) decoding mechanism has emerged as an efficient method. This project aims to comprehensively enhance the reliability of BP-based Polar Code decoding through two approaches: scheduling optimization and structural diversity.

Research Methods

Pruning Algorithm for LDPC-like Sparse Parity-Check Matrix Generation

In our research, we implement the pruning algorithm proposed in [1] and transform $\mathcal{P}(2048, 1024)$ into a sparse parity-check matrix $\mathbf{H}_{pruned}$ with size 3865×4889.

Algorithm 1 Prune **H**-matrix

```
Input:
    $\mathbf{H}_{orig}$.                                   ▷ original H-matrix
Output:
    $\mathbf{H}_{pruned}$.                                 ▷ pruned H-matrix

$\mathbf{H}_{pruned} \leftarrow \mathbf{H}_{orig}$
$\mathbf{H}_{pruned} \leftarrow$ remove_Frozen_VN($\mathbf{H}_{pruned}$)
while true do
    $\mathbf{H}_{pruned} \leftarrow prune\_degree1\_CN(\mathbf{H}_{pruned})$
    $\mathbf{H}_{pruned} \leftarrow condense\_degree1\_VNCH(\mathbf{H}_{pruned})$
    $\mathbf{H}_{pruned} \leftarrow prune\_degree1\_VNH(\mathbf{H}_{pruned})$
    $\mathbf{H}_{pruned} \leftarrow condense\_degree2\_VNH(\mathbf{H}_{pruned})$
    $\mathbf{H}_{pruned} \leftarrow condense\_degree2\_CN(\mathbf{H}_{pruned})$
    if size($\mathbf{H}_{pruned}$) does not change then
        return $\mathbf{H}_{pruned}$
    end if
end while
```

Hybrid RBP

We combine the two RBP decoder in [2], Q-RBP and SVNF-RBP, into a hybrid RBP. The decoding uses a two-stage approach:

- **Stage 1:** The SVNF-RBP decoder runs for 100 iterations to preempt silent node formation.
- **Stage 2:** If Stage 1 fails, the Q-RBP decoder takes over for an additional 100 iterations to disrupt any potential greedy groups.

Algorithm 2 Proposed Hybrid RBP

```
1. Stage 1: Run SVNF-RBP for 100 iterations
2. Initialize all $m_{c\to v} = 0, I = 0$
3. Initialize all $L_{v_j \to c_i} = L_j$
4. Generate $m^{pre}_{c_i \to v_j}$ using (8), $i \in \{1 \cdots M\}, v_j \in \mathcal{N}(c_i)$
5. Compute all $r_{c_i \to v_j}$ using (7)
6. for iteration = 1 to 100 do
7.     for every $v_j$ do
8.         Find $r(m_{c_{max} \to v_{max}}) = \max\{r(m_{c_a \to v_k}) | c_a \in \mathcal{N}(v_j),\ v_k \in \mathcal{N}(c_a),\ k \neq j\}$
9.         Generate $m_{c_{max} \to v_{max}}$ using (4), and propagate
10.        Set $r(m_{c_{max} \to v_{max}}) = 0$
11.        for every $c_a \in \mathcal{N}(v_{max}) \backslash c_{max}$ do
12.            Generate $L_{v_{max} \to c_a}$ using (5), and propagate
13.            for every $v_b \in \mathcal{N}(c_a) \backslash v_{max}$ do
14.                Generate $m^{pre}_{c_a \to v_b}$ using (8)
15.                Compute $r_{c_a \to v_b}$ using (7)
16.            end for
17.        end for
18.    end for
19.    if The stopping rule is not satisfied then
20.        $I = I + 1$
21.        Position = 7
22.    end if
23. end for
24. if The stopping rule is not satisfied then
25. Stage 2: Immediately switch to Q-RBP without initialization
26.     Build max-heap with all edges using $r_{c_i \to v_j}$ from Phase 1
27.     Initialize all $\eta_{c \to v} = 0$
28.     for iteration = 1 to 100 do
29.         Pop edge with $r(m_{c_{max} \to v_{max}}) = \max\{r(m_{c_i \to v_j}) | i \in 1 \cdots M, v_j \in \mathcal{N}(c_i), \eta_{c_i \to v_j} < Q_F + I\}$
30.         Generate $m_{c_{max} \to v_{max}}$ using (4), and propagate
31.         Set $r(m_{c_{max} \to v_{max}}) = 0$ and $\eta_{c_{max} \to v_{max}} = \eta_{c_{max} \to v_{max}} + 1$
32.         for every $c_a \in \mathcal{N}(v_{max}) \backslash c_{max}$ do
33.             Generate $L_{v_{max} \to c_a}$ using (5), and propagate
34.             for every $v_b \in \mathcal{N}(c_a) \backslash v_{max}$ do
35.                 Generate $m^{pre}_{c_a \to v_b}$ using (8)
36.                 Compute $r_{c_a \to v_b}$ using (7)
37.             end for
38.         end for
39.         if The stopping rule is not satisfied then
40.             $I = \left\lceil \frac{\sum_{i=1}^{M} \sum_{v_j \in \mathcal{N}(c_i)} \eta_{c_i \to v_j}}{E} \right\rceil$
41.             Position = 29
42.         end if
43.     end for
44. end if
```

Check-Node Degree Sequence Based Sparse Graph List Decoding

The design concept comes from [3] and [4]. Instead, we use **CN degree sequence** as the fingerprint of each sparse graph (total 500) and applied **PCA** (Principal Components Analysis) and **K-means** algorithm from machine learning as the methods to construct sparse graph list (total 32) for **parallel decoding**.

For each sparse graph $\mathbf{H}_i$:
CN degree sequence $\boldsymbol{d}_{c,i} = \left[\sum_{k=1}^{4889}(\mathbf{H}_i)_{k,1}, \sum_{k=1}^{4889}(\mathbf{H}_i)_{k,2}, \dots, \sum_{k=1}^{4889}(\mathbf{H}_i)_{k,3865}\right]$

The characteristic matrix $\mathbf{D}$ constructed from $\boldsymbol{d}_{c,i}$ of 500 $\mathbf{H}_i$ is therefore:

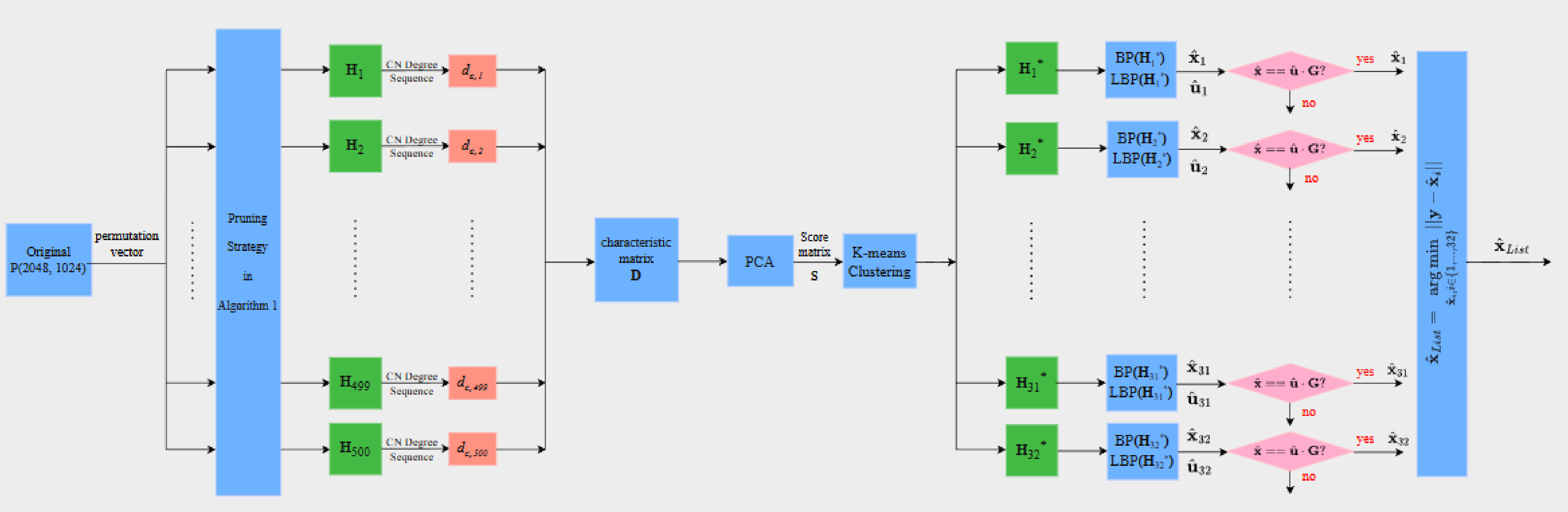
$$\mathbf{D} = \left[\boldsymbol{d}_{c,1}^T, \boldsymbol{d}_{c,2}^T, \dots, \boldsymbol{d}_{c,500}^T\right]^T$$

PCA projects the data from $\mathbf{D}$ onto 50 principal components $\boldsymbol{W}_{50}$ to extract dominant structural patterns $\mathbf{S}$:

$$\mathbf{S} = (\mathbf{D} - \mathbf{1} \cdot \boldsymbol{\mu}^T) \cdot \boldsymbol{W}_{50}$$

K-means partitions 500 sparse graphs $\mathbf{H}_i$ into 32 clusters and select a representative graph for each cluster:

$$\mathcal{J}(\mathcal{C}) = \sum_{j=1}^{32} \sum_{\boldsymbol{s}_i \in \mathcal{C}_j} \left\|\boldsymbol{s}_i - \boldsymbol{\mu}_j\right\|^2, \mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_{32}\}\ ;\ \ i_j^* = \arg\min_{i : \boldsymbol{s}_i \in \mathcal{C}_j} \left\|\boldsymbol{s}_i - \boldsymbol{\mu}_j\right\|^2$$



Simulation Results

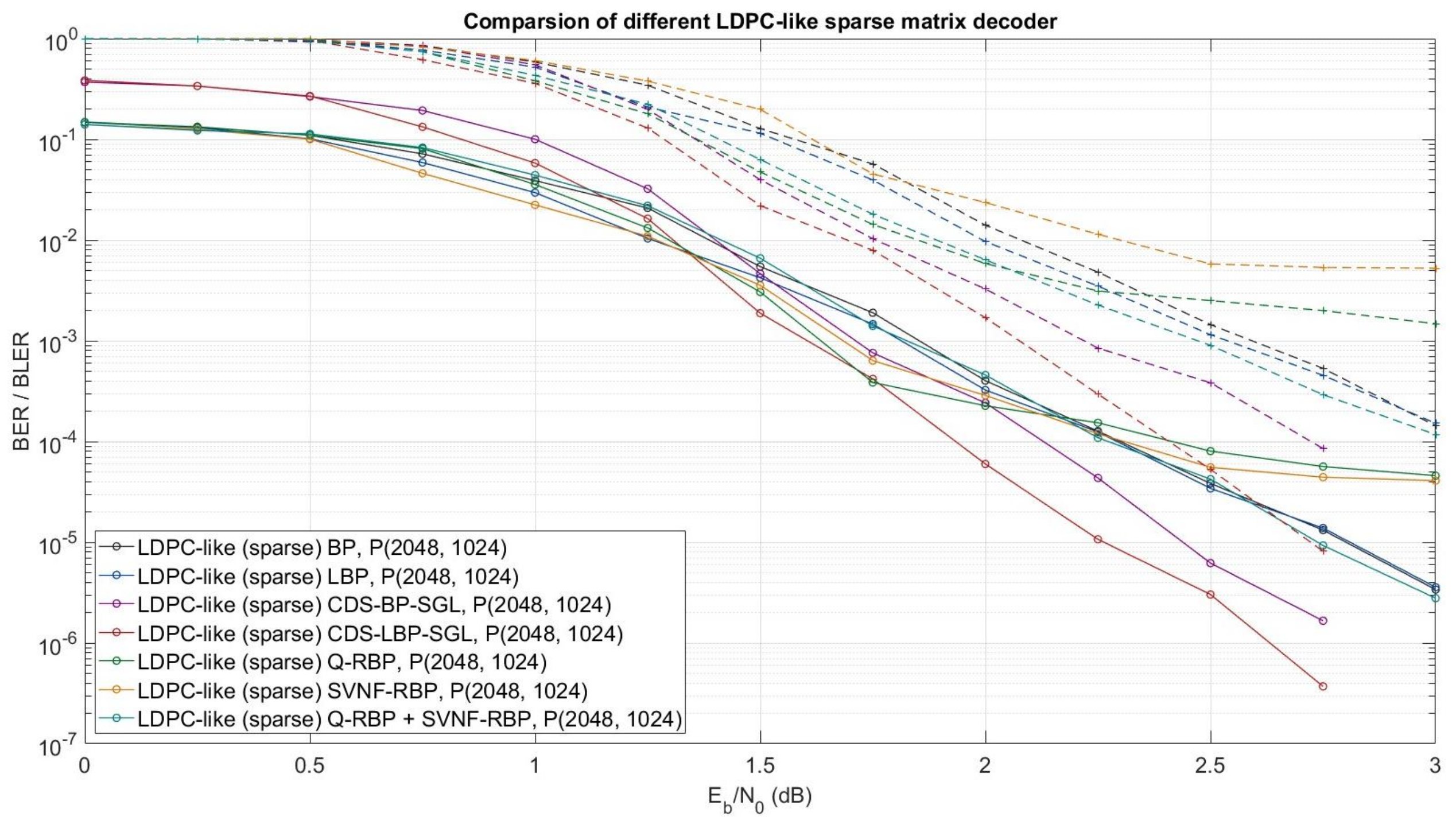


Fig 1. Comparison of BER / BLER for all decoders in this study using the pruned sparse parity-check matrix $\mathbf{H}_{\mathbf{pruned}}$; $N_{it,max} = 200$

Hybrid RBP Decoder Performance

In terms of BER, the Hybrid RBP decoder performed similarly to the standard BP decoder. But it demonstrated an advantage in BLER, showing an improvement of up to 0.3dB compared to the standard BP decoder.

CDS-BP-SGL/ CDS-LBP-SGL Decoder Performance

Although the proposed CDS-BP-SGL and CDS-LBP-SGL decoders showed worse BER and BLER performance than traditional BP and LBP decoders at low and medium SNR (0-1.5 dB), both CDS-SGL algorithms showed significant gains at high SNR (2-3 dB), where there existed an improvement of approximately 0.4-0.6 dB in BER and 0.3-0.5 dB in BLER.

Overall Comparison

The CDS-BP-SGL and CDS-LBP-SGL decoders ultimately outperformed the Hybrid RBP decoder. This is because the Hybrid RBP decoder, despite its optimization, is still limited by the intrinsic structure and inherent trapping sets of a single sparse graph. Conversely, the CDS-SGL decoders, by using multiple graphs with different structures, fundamentally avoid the decoding deadlocks of a specific structure, making their performance gain significantly better than the scheduling optimization gain achieved by the Hybrid RBP decoder.

Conclusion

In this project, we constructed a Hybrid-RBP decoder by integrating Q-RBP and SVNF-RBP, successfully resolving convergence instability without increasing complexity. Furthermore, we proposed the CDS-SGL algorithm for fixed-schedule decoders such as BP and LBP. By utilizing structurally diverse sparse graphs, this method circumvents inherent decoding failures and significantly boosts high-SNR performance. However, the linear growth in hardware complexity and performance degradation in low-to-medium SNR regions remain key challenges, necessitating further optimization in future research.

Reference

[1] S. Cammerer, M. Ebada, A. Elkelesh and S. ten Brink, "Sparse Graphs for Belief Propagation Decoding of Polar Codes," 2018 IEEE International Symposium on Information Theory (ISIT), Vail, CO, USA, 2018, pp. 1465-1469
[2] H. -C. Lee, Y. -L. Ueng, S. -M. Yeh and W. -Y. Weng, "Two Informed Dynamic Scheduling Strategies for Iterative LDPC Decoders," in IEEE Transactions on Communications, vol. 61, no. 3, pp. 886-896, March 2013
[3] A. Elkelesh, M. Ebada, S. Cammerer and S. ten Brink, "Belief Propagation List Decoding of Polar Codes," in IEEE Communications Letters, vol. 22, no. 8, pp. 1536-1539, Aug. 2018
[4] H. Liu, E. Gunawan, H. Yaoyue and Y. L. Guan, "BP-Based Sparse Graph List Decoding of Polar Codes," in IEEE Communications Letters, vol. 27, no. 5, pp. 1257-1261, May 2023