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Sorting Unsigned Sequence by Symmetric Reversals

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Abstract.

We investigate the *decision* problem concerning symmetric reversal operations on unsigned sequences, presenting a linear-time algorithm to determine if it is possible to transform one sequence into another. We also show that corresponding optimization problem, which computes the fewest number of operations for the transformation, is NP- hard.

Problem Definition

Reversal: an operation that reverses the order of a substring of a sequence.
Symmetric reversal: a kind of reversal requires that the substring begins and ends with the same integer. For instance, by asymmetric reversal, we can transform [0, 1, <u>2, 4, 4, 5, 2</u>, 4, 0] into [0, 1, <u>2, 5, 4, 4, 2</u>, 4, 0].
Neighbor: Given a sequence S, we define a neighbor to be an unordered pair of adjacent elements in S.
Neighbor set: The multi-set of all neighbors is called the neighbor set N(S) of S. For instance, the neighbor set of the sequence π = [0, 2, 1, 2, 0] is N(π) = { {0, 2}, {2, 1}, {1, 2}, {2, 0} } . (Note that neighbor is an unordered pair, so we will consider {2, 1} and {1, 2} in the above example as the same.)
Related: Two sequences π and τ are related if and only if N(π) = N(τ). For instance, π = [0, 1, 2, 4, 3, 1, 2, 3, 0] and τ = [0, 1, 2, 3, 4, 2, 1, 3, 0] are related.

In this paper, we are interested in the following decision problem: Given two unsigned sequences π and τ , is it possible to transform π to τ by a series of symmetric reversals?

Surprisingly, the answer is very simple, as stated in the following theorem:

Theorem 1. *Two unsigned sequences can be transformed from one to an other by symmetric reversals if and only if they are related.*

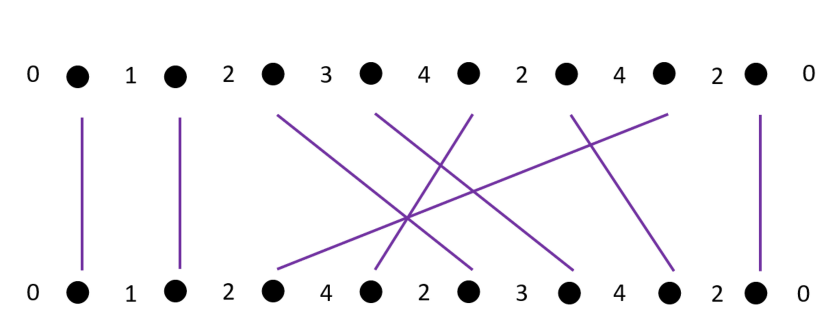
Furthermore, we also consider the more natural optimization problem of finding the fewest number of symmetric reversals needed to transform a sequence to another. Unfortunately, this may not be easy, as suggested by the following theorem.

Theorem 2. *Finding the fewest number of symmetric reversal operations to transform one sequence to another is NP-hard.*

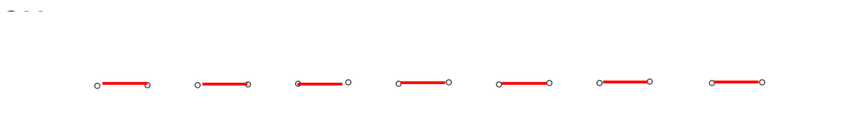
Breakpoint Graph.

We propose a method of constructing a breakpoint graph for any two related sequences.

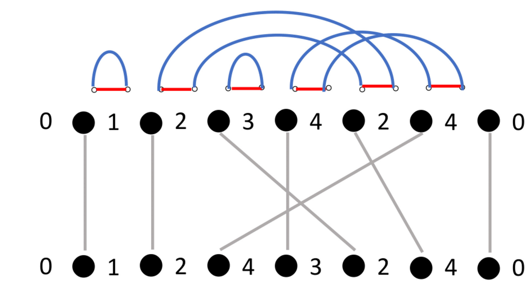
1. We map the same neighbor from τ to π in an arbitrary order (in a one to-one form)



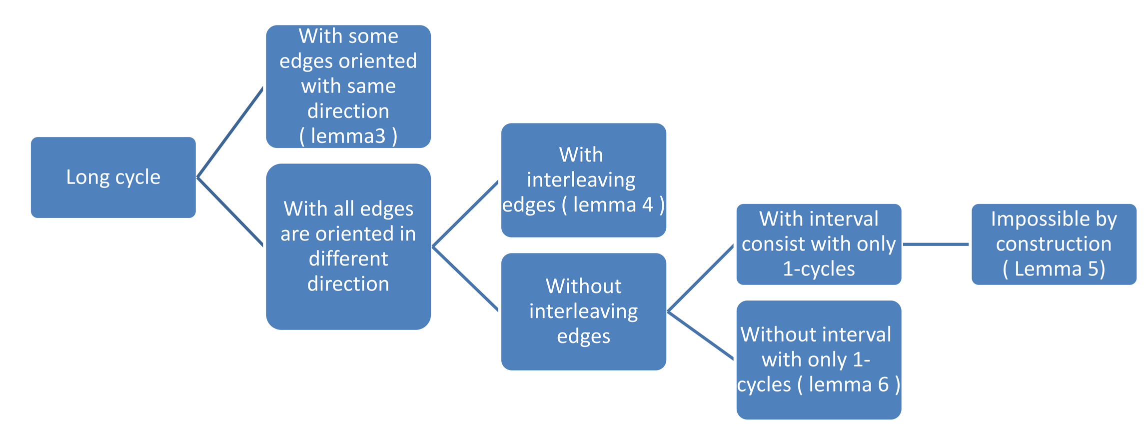
2. For each element except 0, two vertices are extended upwards to facilitate edge connection. Specifically, for each such element, we create two upward-extending vertices: the first is designated as the left vertex, and the second as the right v



3. Connect the extended two points and form red edges (n-1 red edges).
4. For any two adjacent neighbor for τ , connect the same element by a blue edge in π. (total n-1 blue edges).



This is an example of a breakpoint graph.



Goal: Reduce Long Cycle (cycle num ≥ 2)

Lemmas , propositions and theorem

- **Lemma 1:** *Any symmetric reversal will not change the neighbor set.*
- **Lemma 2:** *For any symmetric reversal, it will not increase or decrease the k-cycle inside.*
- **Proposition 1:** *For symmetric reversal on any long cycle, it will only affect the cycle number of itself.*
- **Lemma 3:** *Whenever a long cycle contains edges oriented in the same direction, it is possible to create a 1-cycle using a single symmetric reversal.*
- **Proposition 2:** *If there are no interleaving edges in the long cycle, all the edges must be oriented in opposite directions.*
- **Lemma 4:** *For any long cycle with edge interleaving, it is assured that a series of reversals exists which can effectively decrease the cycle number of that particular long cycle.*
- **Lemma 5:** *In any interval of a long cycle, it cannot exclusively comprise 1-cycles.*
- **Lemma 6:** *For any interval from the long cycle, if there's another long cycle is interleaved within it, reversals can be performed on these interleaving long cycles to ensure that the original long cycle's internal edges exhibit interleaving.*
- **Theorem 1:** *We can transform any two unsigned sequences from one to another by symmetric reversal if and only if they are related.*

NP-Hardness proof.

We know that the minimum distance the sorting by reversals (SBS) for unsigned permutations is NP-hard. Given any permutation e.g. π = [1, 3, 4, 5, 2] we can always add the none existing element k between any two adjacent elements and at both ends making τ = [0, 1, 0, 3, 0, 4, 0, 5, 0, 2, 0].

Through this transformation, as any symmetric reversal does not alter the position of the elements being added, the minimum distance remains unchanged.

This leads to the conclusion that optimization for sorting through symmetric reversal in an unsigned sequence is NP-Hard.

Theorem 2. *Finding the fewest number of symmetric reversal operations to transform one sequence to another is NP-hard.*

Conclusion.

In this paper, we have shown that two sequences can be transformed to one another by symmetric reversals if and only if the two sequences are related, and the decision can be made in O(n) time. However, when minimum distance is concerned, we can show that sorting by symmetric reversals for unsigned sequence is NP-hard. It is open whether a good approximation algorithm exists for this.